Nambu-Poisson manifolds and associated *n*-ary Lie algebroids



2001 J. Phys. A: Math. Gen. 34 9753

(http://iopscience.iop.org/0305-4470/34/45/501)

J. Phys. A: Math. Gen. **34** (2001) 9753 www.iop.org/Journals/ja PII: S0305-4470(01)25072-0

# Corrigendum

**Nambu–Poisson manifolds and associated $n$-ary Lie algebroids**
J A Vallejo 2001 *J. Phys. A: Math. Gen.* **34** 2867–2881

There is an incorrect result in this paper, labelled theorem 9. In the proof it is stated that, by the Leibniz property, it is necessary to check only that

$$[\mathcal{L}_P, [[\ldots[[\mathcal{L}_P, a_1], a_2], \ldots], a_{n-1}]] \in \mathcal{D}_{n-1}$$

when the $a_i's$ are of the form $f \in C^\infty(M)$, $\mathrm{d}f \in \Omega^1(M)$. But this is incorrect. If we take, say, $a_1 = f_1\,\mathrm{d}g_1$, then other terms with a different structure appear, which cannot be cancelled by the reasoning given in the paper. In fact, one might wonder if a different proof can be provided, such as $[\mathcal{L}_P, [[\ldots[[\mathcal{L}_P, a_1], a_2], \ldots], a_{n-1}]] \in \mathcal{D}_{n-1}$ when the $a_i$'s are arbitrary 1-forms, but this is not possible. From the definition of the bracket $[\![., \ldots, .]\!]_{\mathcal{L}_P}$, particularized to the $n = 3$ case for convenience, it is possible to obtain the relation

$$[\![\alpha_1, \alpha_2, \alpha_3]\!]_{\mathcal{L}_P} = \mathrm{d}P(\alpha_1, \alpha_2, \alpha_3) + \mathrm{i}(q(\alpha_2 \wedge \alpha_3))\mathrm{d}\alpha_1 - \mathrm{i}(q(\alpha_1 \wedge \alpha_3))\mathrm{d}\alpha_2 + \mathrm{i}(q(\alpha_1 \wedge \alpha_2))\mathrm{d}\alpha_3$$

and this is the bracket considered by Vaisman [1] and by Grabowski and Marmo [2], which is known to work only with exact 1-forms. This relation holds for any $n$, so the bracket proposed by the author cannot induce any $n$-ary Lie algebroid using the operator $\mathcal{L}_P$.

However, the author believes that the previous results leading to the incorrect theorem 9 are still correct. In particular, they leave open the question of whether there is an operator $D$, different from $\mathcal{L}_P$, which can verify the conditions appearing in theorem 6, namely

$$[D, [[\ldots[[D, a_1], a_2], \ldots], a_{n-1}]] \in \mathcal{D}_{n-1}$$

although, at the time of writing this note, the author has not been able to find a suitable one.

On the other hand, the results contained in section 6, are still valid, as they use the bracket $[\![., \ldots, .]\!]_{\mathcal{L}_P}$ only on exact 1-forms.

The coincidence of the bracket $[\![., \ldots, .]\!]_{\mathcal{L}_P}$ with that proposed by Vaisman was very kindly communicated to the author by M E Padrón and J C Marrero.

## References

[1] Vaisman I 1998 Nambu–Lie groups *Preprint* arXiv math.DG/9812064
[2] Grabowski J and Marmo G 2000 On Filippov algebroids and multiplicative Nambu–Poisson structures *Diff. Geom. Appl.* **12** 35–50